

Full coherent frequency conversion between two microwave propagating modes

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We demonstrate full frequency conversion in the microwave domain using a Josephson three-wave mixing device pumped at the difference between the frequencies of its fundamental eigenmodes. By measuring the signal output as a function of the intensity and phase of the three input signal, idler and pump tones, we show that the device functions as a controllable three-wave beam-splitter/combiner for propagating microwave modes, in accordance with theory. Losses at the full conversion point are found to be less than 10^{-2} . Potential applications of the device include quantum information transduction and realization of an ultra-sensitive interferometer with controllable feedback.

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A quantum information transducer capable of converting the frequency of a quantum signal without introducing noise is one of the desirable modules in quantum communication [1, 2]. With such a device, one could teleport quantum superpositions of ground and excited states of qubits from one system to another one with a different transition frequency, without loss of coherence. If a quantum signal can be routed through different frequency channels, optimization of quantum calculation and communication can be advantageously separated.

The simplest scheme for performing frequency conversion is pumping a dispersive medium with nonlinearity χ_2 at precisely the frequency difference between the input and output frequencies [3–10]. Such nonlinear process known as parametric frequency conversion holds the promise of converting the frequency in a unitary, therefore noiseless, manner, namely, without adding loss or dephasing to the processed signal [11]. In practice however, conversion in the optical domain can be unitary only in the limit of very small converted portion of the input signal. This is due to conversion losses associated with the nonlinear wave mixing process, such as finite interaction lengths and imperfect phase-matching. Similarly, resistive elements used in mixers in the microwave domain, such as Schottky diodes inevitably lead to unacceptable conversion losses. This raises the question as to whether a unitary full conversion of an input signal can be realized practically.

In this work, we show that such noiseless full conversion is possible in the microwave domain by operating a dissipationless three-wave mixing element known as the Josephson parametric converter (JPC) [12–14], in a regime of conversion without photon gain. The JPC performs frequency conversion from 8 to 15 GHz with losses below 0.05 dB at full conversion, as opposed to a typical loss of 6 dB in microwave mixers. We reveal the unitary nature of the JPC conversion by utilizing wave interference between the three incommensurate frequencies intervening the device. The coherence of the conversion process is thus verified without having to face the challenge of calibrating the transmission between different input and output lines of disparate frequencies.

To introduce the requirements obeyed by the properties of unitary frequency conversion, we compare, in Fig. 1, a 3-wave coherent converter and a 2-wave coherent beam-splitter. Panel (a) depicts the desired case of unitary full frequency conversion of an incident propagating beam, i.e. a violet (high frequency) to a red (low frequency) beam, obtained by pumping a dispersive χ_2 medium with a yellow beam whose frequency is precisely the frequency difference. In panel (b) we depict the 2-wave analogue of such a device, that is the total reflector (mirror) whose input and output beams have the same frequency and magnitude but propagate in different directions. Panels (c) and (e) depict the same 3-wave coherent converter shown in panel (a), now operated at the 50/50 beam-splitting point. At this point, half of the power of input beams (red and violet) is transmitted

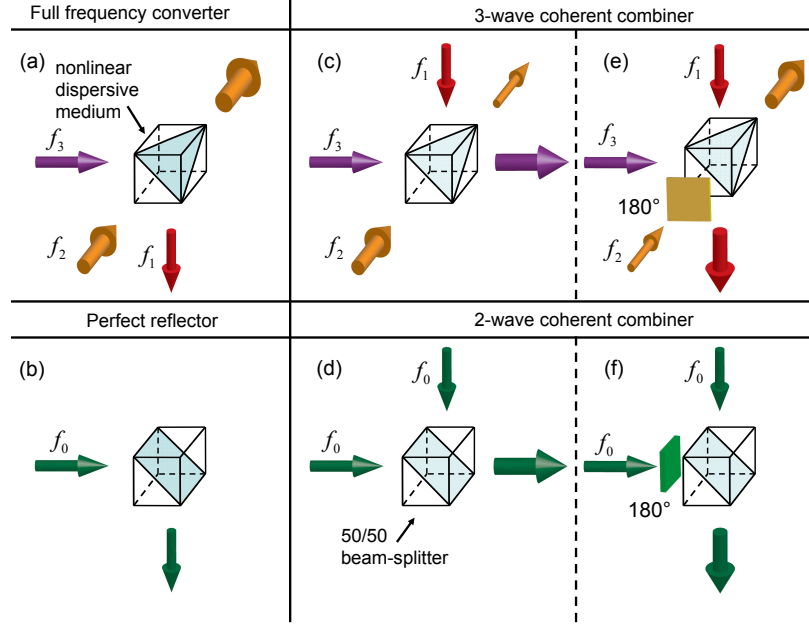


FIG. 1: (color online). In a 3-wave coherent converter, the scattered beams have different frequencies which we indicate using different colors: violet, red and yellow. Panel (a) depicts a nonlinear χ_2 dispersive medium operated as a full frequency converter. An incident violet beam is entirely converted into an output red beam via energy exchange with the incident yellow beam playing here the role of the pump. Panel (b) depicts a splitter modified into a perfect mirror which constitutes the 2-wave analogue of the full frequency converter. Panels (c) and (e) introduce the relationship between frequency conversion and 3-wave interference. By operating the device, shown in panel (a), as a 3-wave 50/50 beam-combiner, one can coherently interfere two beams (red and violet) with disparate frequencies via frequency conversion through the third beam (yellow). Whether a violet beam (panel (c)) or a red beam (panel (e)) is generated at the output depends on the precise setting of all three relative phases of the incident beams. Conversely, in a 2-wave coherent beam-splitter/combiner, interference takes place (panels (d) and (f)) when two equal intensity beams with the same frequency are incident at the beam-splitter. The intensity of the outgoing beams depend on the phase difference between the incoming waves.

while the other half undergoes frequency conversion, again using the third yellow beam. As can be seen in panels (c) and (e), either destructive or constructive 3-wave interference can appear at the output ports of the device depending on the relative phases of the two equal coherent incident red and violet beams. More specifically, panel (c) (panel (e)) describes a destructive interference scenario for the output red (violet) beam. Panels (c) and (e) also indicate the notions that, (1) the output beams of such a device depend on the phases of all incident coherent beams, (2) in order for a lossless destructive interference to take place between two frequencies, the power must be

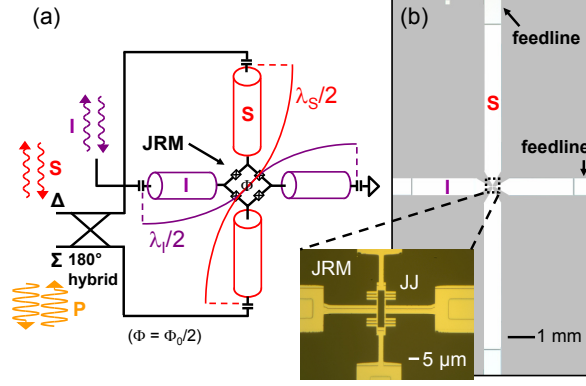


FIG. 2: (color online). Panel (a): circuit diagram of the JPC device. The JPC consists of two half-wave microstrip resonators denoted (S) and (I) which resonate at $f_S = 8.402$ GHz and $f_I = 14.687$ GHz with bandwidths of 95 MHz and 270 MHz respectively. These are set by the coupling capacitors between the resonators and the feedlines. The two resonators intersect at a Josephson ring modulator (JRM), which consists of four nominally identical Josephson junctions arranged in Wheatstone bridge configuration. The JRM is biased near half a flux quantum. The S, I and P coherent waves are represented in this schematic using the same color scheme as Fig. 1 although in this device, $f_P < f_S$. Panel (b): false color optical micrographs of the JPC device and the JRM.

entirely converted to a third frequency. For further clarification of the role of the relative phases, we also show in panels (d) and (f) the 2-wave analogues of the destructive interference scenarios shown in panels (c) and (e).

Akin to the conceptual 3-wave converter depicted in Fig. 1, the JPC is a non-degenerate device with spatial and temporal separation between the idler and signal modes. Its input and output fields share however the same spatial port as shown in Fig. 2 (a) and must therefore be separated using a circulator. The idler and signal modes of the JPC are differential modes of the microstrip resonators of the device which intersect at a Josephson ring modulator (JRM) [13]. An optical micrograph of the JRM, which consists of four Josephson junctions, is shown in Fig. 2 (b). The third mode supported by the device is a non-resonant common-mode drive (pump), whose frequency f_P is set to either the sum of the idler f_I and signal f_S frequencies or their difference. In the case where the pump frequency is equal to the sum $f_P = f_I + f_S$, the device serves as a quantum-limited amplifier which can be used to readout the state of a solid state qubit in real time as has been demonstrated recently [15, 16]. In the present case where the pump frequency verifies $f_P = f_I - f_S$ ($f_I > f_S$), the device operates in the mode of frequency conversion with no photon gain. In this mode, as opposed to amplification [12], the device is not required, according

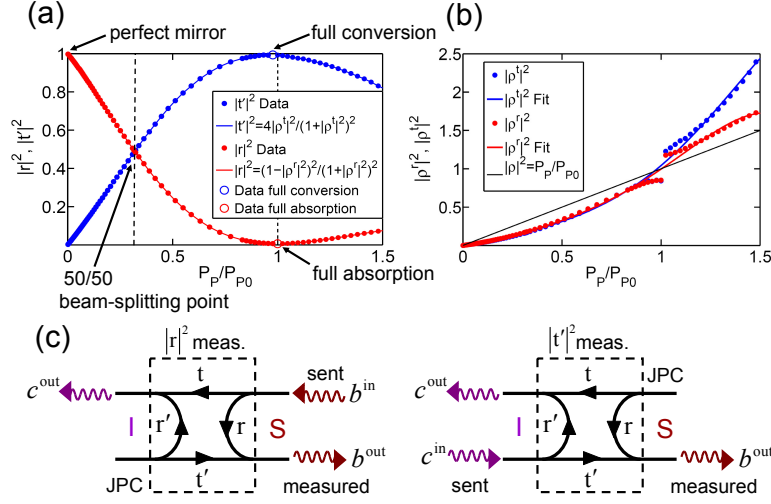


FIG. 3: (color online). (a) Reflection and conversion measurements of the JPC in the conversion mode. The reflection parameter $|r|^2$ and the conversion parameter $|t'|^2$, drawn as filled red and blue circles respectively, are measured at the signal port as a function of the normalized applied pump power P_P/P_{P0} . In the reflection and conversion measurement a signal and an idler tone are applied at f_S and f_I respectively as shown in panel (c). The input photon flux applied in each beam was set to be equal at the 50/50 beam-splitting point as explained in the text. In both measurements, the pump frequency is set to $f_P = f_I - f_S$. The solid red and blue lines correspond to theory expressions. The vertical dashed line on the left (right) indicates the 50/50 beam-splitting (full absorption) working point. In panel (b) we plot using red (blue) circles the relation between the dimensionless parameter $|\rho^r|^2$ ($|\rho^t|^2$) employed in the fits of the reflection (conversion) data (shown in panel (a)) and the normalized pump power applied in the experiment. The solid black line of unity slope corresponds to the expected dependence of $|\rho|^2$ on P_P/P_{P0} based on an ideal model of the JPC and stiff pump condition. The solid red and blue curves are fifth order polynomial fits. (c) Signal flow graphs of the JPC in the conversion mode. In the reflection measurement (shown on the left) a coherent beam is applied to the signal port, whereas in the conversion measurement (shown on the right) an equivalent coherent beam is applied to the idler ports. In both measurements, the output field is measured at the signal port.

to Caves theorem [17] to add any noise. Note that this working regime has been recently the subject of several works in the areas of telecommunication and quantum information processing in optics [4–6, 18–20]. Also, in the recent work done at NIST [8, 9], the authors parametrically converted photons of different frequencies inside a microwave resonator coupled to a dc-SQUID. Phonon-photon parametric frequency conversion is also at play in the active cooling of a micro- or nano- mechanical resonator modes [1, 21, 22]. In our work, by contrast, it is photons from two

different spatial and temporal modes that are interconverted.

When operated in conversion mode and under the stiff pump condition, the JPC can be described as an effective two-port beam-splitter whose scattering parameters can be adjusted by varying the pump tone of the device. In this mode of operation, part of the incoming wave at the idler (signal) port is transmitted to the signal (idler) port after being downconverted (upconverted), via emission (absorption) of pump photons, while the remaining part is reflected off the idler (signal) port.

In Fig. 3 (a) we display measurements of the reflection parameter $|r|^2$ at the signal port (filled red circles) and the idler-to-signal conversion parameter $|t'|^2$ (filled blue circles) as a function of the normalized applied pump power P_P/P_{P0} , where P_{P0} is the pump power at which $|r|^2$ is minimum. The frequency of the pump tone in both measurements is $f_P = 6.285$ GHz. In the reflection measurement, a coherent tone at $f_S = 8.402$ GHz is applied to the signal port with input power $P_{S0} = -123$ dBm, while in the conversion measurement a coherent tone at $f_I = 14.687$ GHz is applied to the idler port (see signal flow graphs of the JPC shown in panel (c)). Both measurements are taken using a spectrum analyzer centered at f_S in zero frequency span mode. The output power measured as a function of P_P/P_{P0} is normalized relative to the reflected signal power obtained with no applied pump power. There, the JPC has unity reflection, which can be measured within ± 0.5 dB accuracy due to a finite impedance mismatch in the output line.

As can be seen in Fig. 3, the splitting ratio between the converted and reflected portions of the beam is set by the device parameters and the pump amplitude. By varying the intensity of the pump, the splitting ratio of the device can be changed continuously from zero conversion and total reflection, to complete conversion and perfect absorption (full converter). It is important to point out that the input power applied to the idler port in the conversion measurement is set to yield, at the 50/50 beam-splitting working point, the same signal output power as the one measured in reflection for P_{S0} . It is remarkable that with this single calibration of input powers, which balances the output at the 50/50 beam-splitting point, the device satisfies quite well the equation of conservation of total photon number $|r|^2 + |t'|^2 = 1$ ($|t| = |t'|$) in the range $P_P/P_{P0} < 1$. When the critical power is traversed $P_P/P_{P0} \gtrsim 1$, we observe a progressive breaking of unitarity $|r|^2 + |t'|^2 < 1$. This can be explained by increased nonlinear effects at elevated pump powers, resulting in frequency conversion to higher modes of the system. However, we find that in the vicinity of P_{P0} , the conversion loss, defined as the deviation of $|r|^2 + |t'|^2$ from unity, to be less than $\pm 10^{-2}$ within $\pm 2.5\%$ accuracy. We also observe a slight shift in power between the peak of the conversion and the minimum of the reflection data (indicated by empty circles). The solid blue

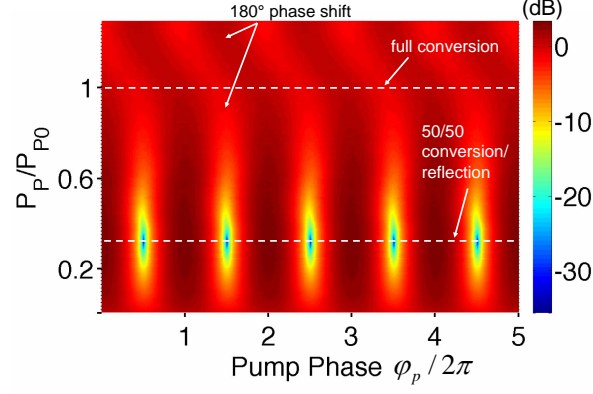


FIG. 4: (color online). Interference fringes of the reflected signal and the converted idler measured at the output of the signal port, at f_S , as a function of pump power and phase. Regions of low contrast correspond to working points with unbalanced splitting ratios, namely, large reflection $|r|^2 \rightarrow 1$ or large conversion $|t'|^2 \rightarrow 1$. The highest contrast, indicated by the bottom dashed white line, signals the 50/50 beam-splitting working point, where $|r|^2 = |t'|^2 = 0.5$. The top dashed white line indicates the full conversion working point where the interference contrast is lowest. For $P_P/P_{P0} > 1$ ($|\rho|^2 > 1$) a phase shift is observed in the interference pattern due to the change in the sign of the reflected signal amplitude, in accordance with theory. The interference power is measured in dB relative to the signal output power with no applied pump and idler tones.

and red curves drawn using the equations shown in the legend of Fig. 3 (a), correspond to theory expressions to the conversion and reflection data respectively (see section III in the supplementary material). In panel (b) we plot, using filled red and blue circles, the dimensionless pump power parameters $|\rho^r|^2$ and $|\rho^t|^2$, which are used to fit the reflection and conversion data as a function of P_P/P_{P0} . The solid black line satisfying the relation $|\rho|^2 = P_P/P_{P0}$, corresponds to the expected dependence of $|\rho|^2$ on P_P/P_{P0} for the ideal JPC model with stiff pump approximation. The solid red and blue curves are fifth order polynomial fits to the $|\rho^r|^2$ and $|\rho^t|^2$ parameters. The fits satisfy three important properties: (1) they pass through the origin ($|\rho^{r,t}|^2 = 0$ for $P_P = 0$), (2) they have a leading linear term in the limit of small pump powers ($P_P \rightarrow 0$), (3) the fits for $|\rho^r|^2$ and $|\rho^t|^2$ coincide for $P_P/P_{P0} < 1$.

In addition to the device dependence on pump power shown in Fig. 3, the relative pump phase plays an important role as well. The downconverted idler and the upconverted signal acquire a phase shift which depends on the pump phase. We utilize this phase dependence as shown in Fig. 4 in order to sensitively interfere signal and idler beams. In this measurement, we applied two coherent tones to the signal and idler ports, having the same frequencies and powers as in the

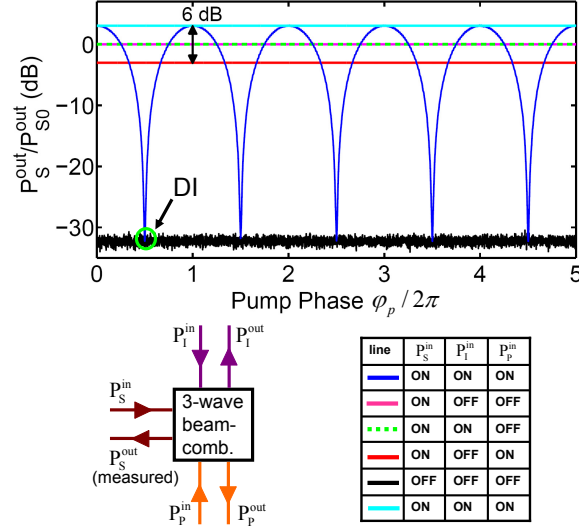


FIG. 5: (color online). The blue curve is a cross-section of the interference fringes shown in Fig. 4 taken at the 50/50 beam-splitting working point. The maximum and minimum points of the curve correspond to constructive and destructive interference (DI) conditions respectively. The magenta line corresponds to the reflected signal power P_{S0}^{out} with no applied pump and idler tones (see the legend table and the device cartoon at the bottom). All curves in the figure are plotted in dB relative to P_{S0}^{out} . The green line corresponds to the relative reflected signal power with applied signal and idler tones (pump off). The red line corresponds to the reflected signal power measured at the 50/50 beam-splitting working point (without idler). The black line corresponds to the relative noise floor of the system set by the HEMT amplifier, measured with no applied tones and as a function of time and not pump phase. The cyan line corresponds to a constructive interference measurement taken for a certain pump phase.

measurement of Fig. 3. We have also phase-locked the three independent coherent waves to the 10 MHz reference oscillator of a rubidium atomic clock. The color mesh in Fig. 4 depicts the interference fringes of the reflected and converted signals generated at the signal port as a function of P_P/P_{P0} and the relative pump phase. As can be seen in the figure, the interference contrast is low for $P_P \ll P_{P0}$ which correspond to almost total reflection. Similarly, the contrast is low for P_P/P_{P0} close to unity which correspond to almost total conversion. A maximum interference contrast is obtained on the other hand for P_P/P_{P0} close to 0.32. We interpret this point as that where our device functions as a 50/50 beam-splitter/combiner. This is further shown in Fig. 5, where we display a cross-section of Fig. 4 at the point of maximum interference contrast. The wave interference modulation curve, shown in blue, is plotted as a function of the relative phase of the pump. The other colored lines represent different reference measurements taken under the same

experimental conditions. The magenta line corresponds to the reflected signal power P_{S0}^{out} obtained without pump or idler tones and forms the reference level for the other measured powers. The dashed green line corresponds to the reflected signal power obtained with no pump but with an equivalent idler tone injected through the idler port. The fact that the dashed green line coincides with the magenta line shows that without pump the idler and signal modes are isolated. The red line represents the reflected signal power at the 50/50 beam-splitting working point and it lies -3 dB below the signal reference line (magenta) as expected. The cyan line corresponds to a maximum constructive interference of the reflected signal and the converted idler, obtained for a certain pump phase. As expected in this constructive interference condition, the relative power level is 6 dB (a factor of 4) above the red line obtained without idler. The black line corresponds to the noise floor measured without any signal. This noise is dominated by the high electron mobility transistor (HEMT) amplifier noise connected at the 4 K stage of our setup. Note that the minimum points of the wave interference curve (blue) correspond to destructive interference (DI) condition and coincide with the HEMT noise floor. By calculating the ratio of the average output power received in a destructive interference experiment to the average power received in a constructive interference experiment, as discussed in section IV in the supplementary material, we get a lower bound (defined for the absolute value) of -37.9 dB on the amount of destructive interference achieved by the device at the 50/50 beam-splitting point. This is equivalent to a coherent cancellation of 99.98% of input signal and idler beams.

Furthermore, in a separate interference measurement in which we varied the input power of the signal and idler beams (see supplementary material), we find that the maximum input power which can be handled by the device is about -100 dBm. Similar to Josephson amplifiers based on the JPC, this figure of merit can be improved by about one order of magnitude, by increasing the critical current of the junctions as discussed in Refs. [13, 14]. We also note that the bandwidth of the present device (~ 70 MHz) is limited by the bandwidths of the resonators. A wider bandwidth, on the order of 500 MHz, could potentially be achieved by substituting the present JRM design with an inductively shunted ring [23].

In addition to quantum information transduction, the device has several potential useful applications, such as cooling of a readout cavity of a qubit by swapping the “hot” cavity photons, for instance at 8 GHz, by “cold” reservoir photons at 15 GHz, and realization of a Mach-Zehnder interferometer (MZI) scheme for microwaves with real-time feedback. In the latter scheme, the device would function as an interferometric 50/50 beam-combiner for incoming idler and signal waves. Assuming, for example, the information is carried by the phase of the signal in the MZI

setup, while the idler wave serves as a reference, the information can be decoded from the incoming signal by measuring the generated interference at the signal port. Moreover, the setpoint of the device, which yields maximum phase sensitivity, can be maintained in-situ by compensating the shift using the pump phase.

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Supplementary Material for “Full coherent frequency conversion between two microwave propagating modes”

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I. IDEAL MIXER EQUATIONS

The Hamiltonian of an ideal 3-wave mixing device with internal modes a , b and c can be written in the framework of the Rotating Wave Approximation (RWA) of the system as,

$$\frac{H_0^{\text{RWA}}}{\hbar} = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c + g_3 (a^\dagger b^\dagger c + abc^\dagger), \quad (\text{S1})$$

where we kept only terms commuting with the total photon number. The operators a^\dagger , b^\dagger , c^\dagger (a , b , c) are the creation (annihilation) operators of modes a , b and c respectively, which satisfy the bosonic commutation relations $[a, a^\dagger] = [b, b^\dagger] = [c, c^\dagger] = 1$. The angular frequencies ω_a , ω_b , ω_c are the resonance frequencies of modes a , b and c , where $\omega_a < \omega_b < \omega_c$ and ω_c satisfies $\omega_c = \omega_a + \omega_b$. The angular frequency g_3 denotes the coupling constant between the modes.

Treating the coupling of each oscillator with a transmission line carrying waves in and out of the oscillator in the RWA, one arrives at three coupled quantum Langevin equations

$$\begin{aligned} \frac{d}{dt}a &= -i\omega_a a - ig_3 b^\dagger c - \frac{\gamma_a}{2}a + \sqrt{\gamma_a}\tilde{a}^{\text{in}}(t), \\ \frac{d}{dt}b &= -i\omega_b b - ig_3 a^\dagger c - \frac{\gamma_b}{2}b + \sqrt{\gamma_b}\tilde{b}^{\text{in}}(t), \\ \frac{d}{dt}c &= -i\omega_c c - ig_3 ab - \frac{\gamma_c}{2}c + \sqrt{\gamma_c}\tilde{c}^{\text{in}}(t), \end{aligned} \quad (\text{S2})$$

where the second term in the right hand side corresponds to the non-linear term producing photon conversion, $\gamma_{a,b,c}$ is the rate at which photons introduced in one resonator leave, and $\tilde{a}^{\text{in}}(t)$, $\tilde{b}^{\text{in}}(t)$, $\tilde{c}^{\text{in}}(t)$ are input fields which correspond to the negative frequency component of the drive terms in the classical equations. Taking for example $\tilde{a}^{\text{in}}(t)$, the input fields are given by

$$\tilde{a}^{\text{in}}(t) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} a^{\text{in}}[\omega] e^{-i\omega t} d\omega, \quad (\text{S3})$$

where $a^{\text{in}}[\omega]$ are the usual field operators obeying the commutation relations

$$[a^{\text{in}}[\omega], a^{\text{in}}[\omega']] = \text{sgn}\left(\frac{\omega - \omega'}{2}\right) \delta(\omega + \omega') \quad (\text{S4})$$

in which ω denotes a frequency that can be positive or negative.

Using the standard input-output relations for the three oscillators given by

$$\sqrt{\gamma_a}a(t) = \tilde{a}^{\text{in}}(t) + \tilde{a}^{\text{out}}(t), \quad (\text{S5})$$

$$\sqrt{\gamma_b}b(t) = \tilde{b}^{\text{in}}(t) + \tilde{b}^{\text{out}}(t), \quad (\text{S6})$$

$$\sqrt{\gamma_c}c(t) = \tilde{c}^{\text{in}}(t) + \tilde{c}^{\text{out}}(t), \quad (\text{S7})$$

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we can express equations (S2) as

$$O_a^+ \tilde{a}^{\text{out}}(t) = -O_a^- \tilde{a}^{\text{in}}(t) - i\sqrt{\gamma_a} g_3 b^\dagger c, \quad (\text{S8})$$

$$O_b^+ \tilde{b}^{\text{out}}(t) = -O_b^- \tilde{b}^{\text{in}}(t) - i\sqrt{\gamma_b} g_3 a^\dagger c, \quad (\text{S9})$$

$$O_c^+ \tilde{c}^{\text{out}}(t) = -O_c^- \tilde{c}^{\text{in}}(t) - i\sqrt{\gamma_c} g_3 ab, \quad (\text{S10})$$

where

$$O_{a,b,c}^\pm = \frac{d}{dt} + i(\omega_{a,b,c} \mp i\Gamma_{a,b,c}), \quad (\text{S11})$$

$$\Gamma_{a,b,c} = \frac{\gamma_{a,b,c}}{2}. \quad (\text{S12})$$

Equations (S8), (S9), (S10) in combination with relations (S5), (S6), (S7) form a set of 6 coupled equations which represent an ideal 3-wave mixing device. Unfortunately, these equations lack an analytical solution in the general case where all modes and drives are treated on the same footing. However, an analytical solution can be found under certain conditions as discussed in the following subsections, namely, if all drives are very small or if one of the drives is stronger than the others.

II. SMALL SIGNAL APPROXIMATION

Fourier transforming equations (S8), (S9), (S10) gives

$$\chi_a^{-1}[\omega_1] a^{\text{out}}[\omega_1] = \chi_a^{-1*}[\omega_1] a^{\text{in}}[\omega_1] - i \frac{2g_3}{\sqrt{\gamma_a}} (b^\dagger[\omega_2] * c[\omega_3]), \quad (\text{S13})$$

$$\chi_b^{-1}[\omega_2] b^{\text{out}}[\omega_2] = \chi_b^{-1*}[\omega_2] b^{\text{in}}[\omega_2] - i \frac{2g_3}{\sqrt{\gamma_b}} (a^\dagger[\omega_1] * c[\omega_3]), \quad (\text{S14})$$

$$\chi_c^{-1}[\omega_3] c^{\text{out}}[\omega_3] = \chi_c^{-1*}[\omega_3] c^{\text{in}}[\omega_3] - i \frac{2g_3}{\sqrt{\gamma_c}} (a[\omega_1] * b[\omega_2]), \quad (\text{S15})$$

where the asterisk “*” stands for convolution operation, $\omega_1, \omega_2, \omega_3$ are the excitation angular frequencies of modes a, b and c respectively, $\chi_{a,b,c}$ is a dimensionless response function whose inverse is given by

$$\chi_{a,b,c}^{-1}[\omega_{1,2,3}] = 1 - i \frac{\omega_{1,2,3} - \omega_{a,b,c}}{\Gamma_{a,b,c}}, \quad (\text{S16})$$

and $a[\omega_1], b[\omega_2], c[\omega_3]$ satisfy the input-output relations in the frequency domain which read

$$\sqrt{\gamma_a} a[\omega_1] = \tilde{a}^{\text{in}}[\omega_1] + \tilde{a}^{\text{out}}[\omega_1], \quad (\text{S17})$$

$$\sqrt{\gamma_b} b[\omega_2] = \tilde{b}^{\text{in}}[\omega_2] + \tilde{b}^{\text{out}}[\omega_2], \quad (\text{S18})$$

$$\sqrt{\gamma_c} c[\omega_3] = \tilde{c}^{\text{in}}[\omega_3] + \tilde{c}^{\text{out}}[\omega_3]. \quad (\text{S19})$$

In case the drives are small $|\langle \tilde{a}^{\text{in}} \rangle|^2 \ll 1$, $|\langle \tilde{b}^{\text{in}} \rangle|^2 \ll 1$, $|\langle \tilde{c}^{\text{in}} \rangle|^2 \ll 1$, we can obtain a first order solution for the internal modes of the system

$$a[\omega_1] = 2\chi_a[\omega_1] \frac{\tilde{a}^{\text{in}}[\omega_1]}{\sqrt{\gamma_a}}, \quad (\text{S20})$$

$$b[\omega_2] = 2\chi_b[\omega_2] \frac{\tilde{b}^{\text{in}}[\omega_2]}{\sqrt{\gamma_b}}, \quad (\text{S21})$$

$$c[\omega_3] = 2\chi_c[\omega_3] \frac{\tilde{c}^{\text{in}}[\omega_3]}{\sqrt{\gamma_c}}, \quad (\text{S22})$$

by neglecting the nonlinear terms on the right-hand side of equations (S13), (S14), (S15) and substituting the resultant output fields in equations (S17), (S18), (S19).

Furthermore, by using equations (S20), (S21), (S22) as perturbation in equations (S13), (S14), (S15) we get

$$\chi_a^{-1}[\omega_1] \tilde{a}^{\text{out}}[\omega_1] = \chi_a^{-1*}[\omega_1] \tilde{a}^{\text{in}}[\omega_1] - i \frac{8g_3}{\sqrt{\gamma_a \gamma_b \gamma_c}} \left(\frac{b^{\text{in}}[-\omega_2]}{\chi_b^{-1*}[\omega_2]} * \frac{c^{\text{in}}[\omega_3]}{\chi_c^{-1}[\omega_3]} \right), \quad (\text{S23})$$

$$\chi_b^{-1}[\omega_2] \tilde{b}^{\text{out}}[\omega_2] = \chi_b^{-1*}[\omega_2] \tilde{b}^{\text{in}}[\omega_2] - i \frac{8g_3}{\sqrt{\gamma_a \gamma_b \gamma_c}} \left(\frac{a^{\text{in}}[-\omega_1]}{\chi_a^{-1*}[\omega_1]} * \frac{c^{\text{in}}[\omega_3]}{\chi_c^{-1}[\omega_3]} \right), \quad (\text{S24})$$

$$\chi_c^{-1}[\omega_3] \tilde{c}^{\text{out}}[\omega_3] = \chi_c^{-1*}[\omega_3] \tilde{c}^{\text{in}}[\omega_3] - i \frac{8g_3}{\sqrt{\gamma_a \gamma_b \gamma_c}} \left(\frac{a^{\text{in}}[\omega_1]}{\chi_a^{-1}[\omega_1]} * \frac{b^{\text{in}}[\omega_2]}{\chi_b^{-1}[\omega_2]} \right). \quad (\text{S25})$$

At resonance tuning $\omega_1 = \omega_a$, $\omega_2 = \omega_b$, $\omega_3 = \omega_c$ where $\chi_{a,b,c}^{-1} = 1$ this solution reduces into

$$\tilde{a}^{\text{out}}[\omega_a] = \tilde{a}^{\text{in}}[\omega_a] - i \frac{8g_3}{\sqrt{\gamma_a \gamma_b \gamma_c}} (b^{\text{in}}[-\omega_b] * c^{\text{in}}[\omega_c]), \quad (\text{S26})$$

$$\tilde{b}^{\text{out}}[\omega_b] = \tilde{b}^{\text{in}}[\omega_b] - i \frac{8g_3}{\sqrt{\gamma_a \gamma_b \gamma_c}} (a^{\text{in}}[-\omega_a] * c^{\text{in}}[\omega_c]), \quad (\text{S27})$$

$$\tilde{c}^{\text{out}}[\omega_c] = \tilde{c}^{\text{in}}[\omega_c] - i \frac{8g_3}{\sqrt{\gamma_a \gamma_b \gamma_c}} (a^{\text{in}}[\omega_a] * b^{\text{in}}[\omega_b]). \quad (\text{S28})$$

Note that the convolution in the second term on the right hand side of equations (S26), (S27), (S28) represents a product of the input fields in the time domain.

III. JPC SCATTERING MATRIX

In this subsection, we derive the scattering matrix of the JPC in the noiseless conversion regime. Assuming that modes a or b are strongly driven, which is the regime of operation discussed in this work, then equations (S8), (S9), (S10) can be solved analytically and yield a scattering matrix of an ideal mixer. To obtain the solution for this case, we assume, without loss of generality, that $\gamma_a \gg \gamma_b, \gamma_c$ and that we pump oscillator a with a strong classical drive. In this case, modes b and c represent the signal and idler modes and a the pump mode. Under these conditions, we can replace operator $a(t)$ by the average value in the coherent state produced by the pump $\langle a(t) \rangle = |\alpha| e^{-i(\omega_p t + \varphi_p)}$, where $|\alpha|$ is the amplitude of the coherent drive, φ_p is the phase of the drive and $\omega_p = \omega_i - \omega_s$, where ω_s, ω_i are the angular frequencies of the signal and idler excitations. Substituting this relation in equations (S8), (S9), (S10) and using equations (S5), (S6), (S7), one gets two equations in the time domain for $\tilde{b}^{\text{out}}(t)$ and $\tilde{c}^{\text{out}}(t)$ as a function of $\tilde{b}^{\text{in}}(t)$ and $\tilde{c}^{\text{in}}(t)$. By Fourier transforming these two equations into the frequency domain, we arrive at the scattering matrix of the JPC in the conversion mode given by

$$\begin{bmatrix} b^{\text{out}}[\omega_s] \\ c^{\text{out}}[\omega_i] \end{bmatrix} = \begin{bmatrix} r_{bb} & t_{bc} \\ t_{cb} & r_{cc} \end{bmatrix} \begin{bmatrix} b^{\text{in}}[\omega_s] \\ c^{\text{in}}[\omega_i] \end{bmatrix}, \quad (\text{S29})$$

where $b^{\text{in}}, c^{\text{in}}$ and $b^{\text{out}}, c^{\text{out}}$ are the incoming and outgoing wave amplitudes of the signal and idler respectively, given in units of square root of photon number per unit frequency, r_{bb}, r_{cc} are the amplitude reflection parameters for the signal and idler and t_{bc}, t_{cb} are the conversion parameters from idler to signal and vice versa, which are given by

$$r_{bb} = \frac{\chi_b^{-1*} \chi_c^{-1} - |\rho|^2}{\chi_b^{-1} \chi_c^{-1} + |\rho|^2}, \quad (\text{S30})$$

$$r_{cc} = \frac{\chi_b^{-1} \chi_c^{-1*} - |\rho|^2}{\chi_b^{-1} \chi_c^{-1} + |\rho|^2}, \quad (\text{S31})$$

$$t_{bc} = \frac{2i\rho}{\chi_b^{-1} \chi_c^{-1} + |\rho|^2}, \quad (\text{S32})$$

$$t_{cb} = \frac{2i\rho^*}{\chi_b^{-1} \chi_c^{-1} + |\rho|^2}, \quad (\text{S33})$$

where the dimensionless response functions $\chi_{b,c}$ and the parameter ρ are given by

$$\chi_{b,c}^{-1} = 1 - i \frac{\omega_{s,i} - \omega_{b,c}}{\Gamma_{b,c}}, \quad (\text{S34})$$

$$\rho = \sqrt{\frac{P_P}{P_{P0}}} e^{-i\varphi_P}, \quad (\text{S35})$$

where $P_P = \hbar\omega_p\gamma_p|\alpha|^2$ is the pump power and $P_{P0} = \hbar\omega_p\gamma_p\Gamma_b\Gamma_c/g_3^2$ where $\gamma_p = \gamma_a$.

Note that the scattering matrix is unitary (the total number of photons is conserved) and satisfies the relations $|r_{bb}|^2 + |t_{cb}|^2 = 1$, $|r_{cc}|^2 + |t_{bc}|^2 = 1$.

For zero frequency detuning, i.e. $\chi_b^{-1} = \chi_c^{-1} = 1$, the scattering matrix parameters reduce to $r = r_{bb} = r_{cc}$, $t = t_{bc} = -t_{cb}^*$ where

$$r = \frac{1 - |\rho|^2}{1 + |\rho|^2}, \quad (\text{S36})$$

$$t = \frac{2i\rho}{1 + |\rho|^2}. \quad (\text{S37})$$

Equivalently, the unitary scattering matrix of the device can be written in the form

$$\begin{bmatrix} \cos \theta & e^{-i\varphi_P} \sin \theta \\ -e^{i\varphi_P} \sin \theta & \cos \theta \end{bmatrix}. \quad (\text{S38})$$

It is worthwhile mentioning that if instead of oscillator a and b discussed above, oscillator c satisfies $\gamma_c \gg \gamma_a, \gamma_b$ and it is driven by a bright classical tone $|\langle c^{\text{in}} \rangle|^2 \gg 1$, which produces a coherent state, then equations (S8), (S9), (S10) and (S5), (S6), (S7) can be solved in a similar manner as was done for modes a and b and yield the scattering matrix of a quantum-limited amplifier (see Ref. [1]).

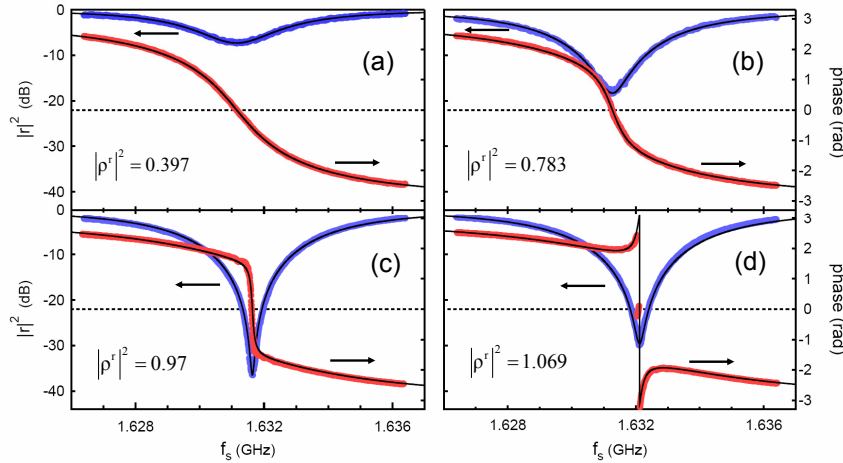


FIG. S1: (colored online). Reflection parameter versus signal frequency. Reflection parameter magnitude (blue curves) and phase (red curves) of a JPC device measured in conversion mode as a function of the signal frequency. The measurements exhibited in subplots (a), (b), (c) and (d) are taken using a vector network analyzer and correspond to different applied pump powers. The black curves are theory fits to the data using Eq. (S30). The values for $|\rho^r|^2$ listed in each panel are extracted from the theory fits. The pump frequency applied in this measurement is $f_P = 5.585$ GHz. The resonance frequency of the idler mode is $f_b = 7.216$ GHz and the quality factors of resonator a (S) and b (I) are $Q_a = 450$ and $Q_b = 120$ respectively.

In Fig. S1 we show a reflection parameter measurement of a JPC operated in conversion mode. The measured device was previously presented in Ref. [2] and operated as a quantum-limited amplifier. Subplots (a), (b), (c) and (d) display reflection parameter data taken for different applied pump powers at $f_P = 5.585$ GHz. The blue and red

curves correspond to the reflection parameter magnitude and phase measured using a vector network analyzer as a function of the device signal frequency. The black curves are theory fits to the data plotted using equation (S30). As expected, the reflection magnitude varies with the pump power. It decreases as the pump power is increased as shown in subplots (a) and (b). It reaches a minimum for a certain pump power P_{P0} corresponding to $\rho^r \simeq 1^-$ as shown in subplot (c), whereas for larger pump powers corresponding to $\rho^r > 1$ it increases again as displayed in subplot (d). Furthermore, as can be seen in subplot (d), the reflection phase undergoes a sign change for $\rho^r > 1$ as predicted by Eq. (S30).

A. Interference using the JPC

In an interference experiment, two equal tones are injected to the signal and idler ports of the JPC, that is $|b^{\text{in}}[\omega_s]|^2 = |c^{\text{in}}[\omega_i]|^2$. The output signal photon number per unit frequency reads

$$|b^{\text{out}}[\omega_s]|^2 = |rb^{\text{in}}[\omega_s] + tc^{\text{in}}[\omega_i]|^2 \quad (\text{S39})$$

$$= \left| |r| + |t| e^{i(\varphi_i - \varphi_s - \varphi_p)} \right|^2 |b^{\text{in}}[\omega_s]|^2, \quad (\text{S40})$$

where φ_s and φ_i are the phases of the input signal and idler coherent signals.

At the 50/50 beam-splitting point, $|r| = |t| = 1/\sqrt{2}$ which yields an interference pattern given by

$$\frac{|b^{\text{out}}[\omega_s]|^2}{|b^{\text{in}}[\omega_s]|^2} = 2 |\cos((\varphi_i - \varphi_s - \varphi_p)/2)|^2, \quad (\text{S41})$$

which depends on the phases of the three coherent beams. When comparing the peaks of these interference fringes to the reflection and conversion parameters at the 50/50 beam-splitting point ($|r|^2 = |t|^2 = 1/2$) we obtain the 6 dB difference observed in the experiment (see Fig. 5 in the main text).

B. Manley-Rowe relations

In a reactive lossless nonlinear device with multiple frequencies such as the JPC, the net flux of photons (number of photons per unit time) of pump ΔN_P , signal ΔN_S and idler ΔN_I participating in the mixing interaction obey the following Manley-Rowe relations

$$|\Delta N_P| = |\Delta N_S| = |\Delta N_I|, \quad (\text{S42})$$

where

$$|\Delta N_P| = |N_P^{\text{in}} - N_P^{\text{out}}|, \quad (\text{S43})$$

$$|\Delta N_S| = |N_S^{\text{in}} - N_S^{\text{out}}|, \quad (\text{S44})$$

$$|\Delta N_I| = |N_I^{\text{in}} - N_I^{\text{out}}|, \quad (\text{S45})$$

and $N_P^{\text{in,out}} = |\tilde{a}^{\text{in,out}}(t)|^2$, $N_S^{\text{in,out}} = |\tilde{b}^{\text{in,out}}(t)|^2$, $N_I^{\text{in,out}} = |\tilde{c}^{\text{in,out}}(t)|^2$. Equation (S42) can be also written in terms of the net power contained in the different modes

$$\left| \frac{\Delta P_P}{\hbar\omega_p} \right| = \left| \frac{\Delta P_S}{\hbar\omega_s} \right| = \left| \frac{\Delta P_I}{\hbar\omega_i} \right|, \quad (\text{S46})$$

where $\Delta P_{P,S,I} = P_{P,S,I}^{\text{in}} - P_{P,S,I}^{\text{out}}$.

In the interference measurement shown in Fig. S2 (a) we set the JPC at the 50/50 beam-splitting working point using the pump tone and set the input signal power to P_{S0} which corresponds to a signal input photon flux N_{S0}^{in} and we vary the idler input photon flux N_I^{in} relative to N_{S0}^{in} by varying the input idler power. As can be seen in the figure, the largest interference contrast induced by the relative pump phase is obtained at 0 dB, at which the input photon

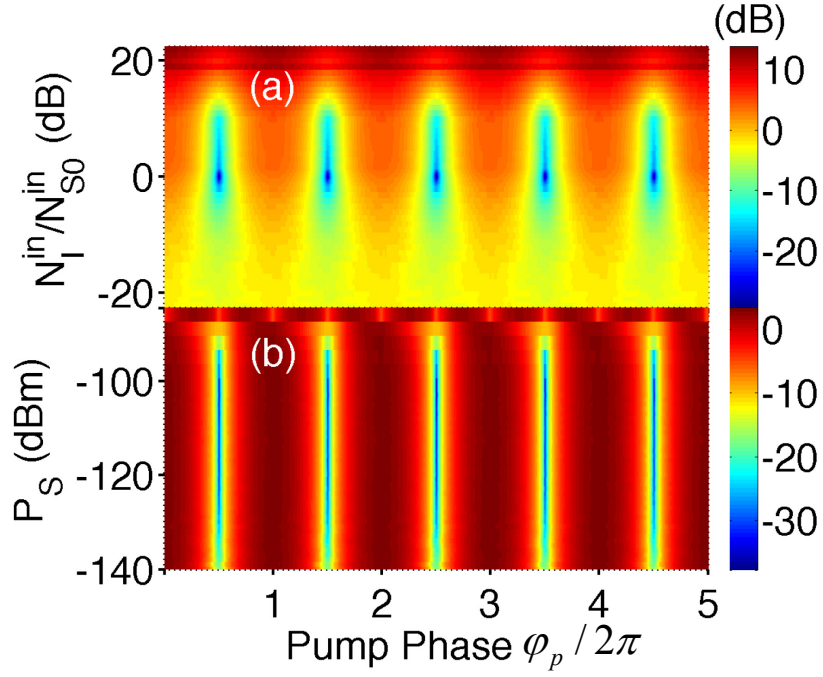


FIG. S2: (colored online). Interference fringes measured at the signal port as a function of the pump phase at the 50/50 beam-splitting working point. The applied S, I and P frequencies are identical to the values cited in the main text. In panel (a) the interference is obtained for a varying idler input photon flux N_I^{in} relative to a constant applied signal photon flux N_{S0}^{in} obtained at P_{S0} . As expected, the highest contrast is achieved for the case of 0 dB, which corresponds to equal I and S photon fluxes. In panel (b) the interference is obtained by injecting equal input S and I photon fluxes ($N_I^{\text{in}} = N_S^{\text{in}}$) and varying the corresponding input powers by the same step. In this color plot we show the generated interference as a function of the corresponding signal input power P_S . As can be seen in the plot, the device maintains its high contrast interference up to a maximum input power of -100 dBm above which the device starts to saturate. At very small input powers below -130 dBm the interference modulation amplitude is limited by the noise floor of the system.

fluxes N_I^{in} and N_{S0}^{in} are equal. In Fig. S2 (b), we set the JPC at the same 50/50 beam-splitting working point, but we inject equal signal and idler photon fluxes and vary the corresponding input powers jointly. This measurement shows the input power range on the signal port P_S over which the device is capable of maintaining its interference contrast before it saturates due to nonlinear effects or pump depletion [1]. As can be seen in the figure, the maximum input power which can be handled by the device is about -100 dBm.

C. Electromagnetic Induced Transparency

It is worthwhile pointing out the strong similarity that exists between parametric frequency conversion in our Josephson device and the electromagnetic induced transparency (EIT) effect in optics [3–6]. In the EIT effect, the optical properties (i.e. absorption) of a 3-level atomic medium, as seen by a weak probe signal that is in resonance with one of the atomic transitions, are modified by driving the system with a strong coherent Rabi field in resonance with another atomic transition. In such a 3-level system, the Rabi drive facilitates a quantum interference between two pathways for the impinging probe field. As the interference becomes destructive for a certain drive amplitude, absorption vanishes and the medium becomes transparent. Likewise, the JPC in conversion mode can be thought as a 3-level atomic medium whose microwave properties (i.e. total reflection), as seen by a weak signal, can be changed to perfect absorption (i.e. full conversion) by inducing destructive interference in the device using a strong Rabi field that is the pump. Hence, the reflection parameter expressed in Eq. (S30) is in a way analogous to the complex susceptibility of the medium in the EIT effect.

IV. COHERENT CANCELLATION OF OUTPUT BEAMS

In order to quantify the amount of coherent cancellation achieved by the device in a DI experiment at the 50/50 beam-splitting point, we measure the received rms voltage distribution of the system noise with no applied tones, which is set by the output amplification chain, and measure also the received rms voltage distribution of the signal in a DI condition with three applied coherent tones as indicated by the green circle in Fig. 5 in the main text. In the DI case the relative pump phase was manually set using a phase shifter to yield a maximum destructive interference. In Fig. S3 we plot the measured noise and DI distributions normalized by the total number of points using black stars and blue circles respectively. Both distributions are constructed using averaged time traces of a spectrum analyzer operated in zero frequency span mode at f_S . As indicated in Fig. S3, the data is taken for two resolution bandwidths $B_1 = 10\text{ kHz}$ and $B_2 = 510\text{ Hz}$ drawn on the right and left sides of the plot. The dashed black curve and the dashed blue curve are Gaussian fits to the data. The Gaussian means of the noise distributions shown in Fig. S3, $\mu_{\text{rms}}^{\text{N1,fit}} = 10.08 \cdot 10^{-5}\text{ V}$ for B_1 and $\mu_{\text{rms}}^{\text{N2,fit}} = 2.3 \cdot 10^{-5}\text{ V}$ for B_2 agree well with the calculated rms voltage of thermal noise generated by a $R = 50\ \Omega$ load connected to a $50\ \Omega$ transmission line and amplification stages, i.e. HEMT and room temperature amplifiers, given by $\mu_{\text{rms}}^{\text{N1,calc}} = \sqrt{gk_B T_N B_1 R} = 9.8 \cdot 10^{-5}\text{ V}$ and $\mu_{\text{rms}}^{\text{N2,calc}} = 2.2 \cdot 10^{-5}\text{ V}$, where $g = 81 \pm 2\text{ dB}$ is the gain of the signal output chain, k_B is Boltzmann constant and $T_N = 11 \pm 2\text{ K}$ is the noise temperature of the output signal line.

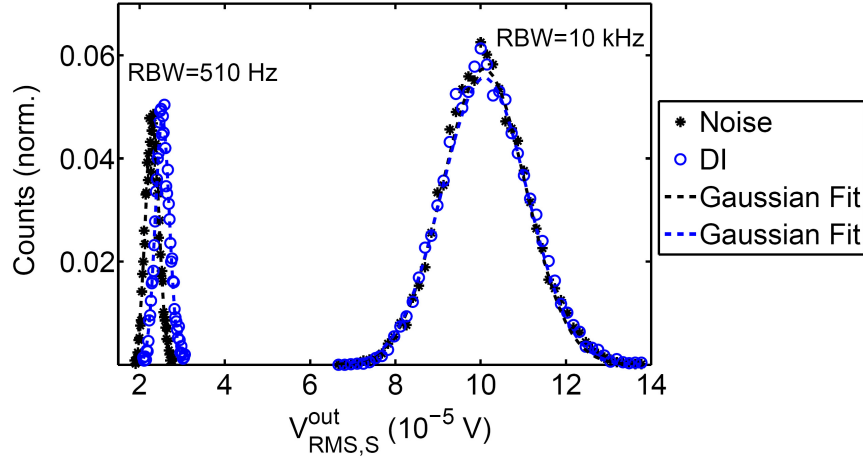


FIG. S3: (colored online). Normalized histograms of received rms voltage measured for the system noise (black stars) and for the destructive interference (DI) experiment (blue circles). The noise and DI histograms, normalized by the total number of points, are taken using a spectrum analyzer operated in zero frequency span mode at $f_S = 8.402\text{ GHz}$. The different distributions for the noise and DI plotted on the right and left hand side are measured using two resolution bandwidths 10 kHz and 510 Hz respectively. In the case of the noise measurement, no tone is applied, while in the case of the DI, the device is operated at the 50/50 beam-splitting working point and three continuous wave tones are applied simultaneously: signal, idler and pump. The parameters of the three applied tones are identical to the values used in the measurement shown in Fig. 5. In this measurement, the pump phase is manually set to a relative phase which maximizes the DI. The DI working point is indicated by a green circle in Fig. 5. The histograms on the right (left) are constructed using 10^4 ($5 \cdot 10^3$) points averaged over 30 (100) traces with a sweep time of 10 s (1 s). The traces were measured with equal resolution and video bandwidths of 10 kHz (510 Hz). The black and the blue dashed lines are Gaussian fits to the noise and DI measurements respectively.

As can be seen in the figure, the noise and DI distributions lie on top of each other in the first measurement corresponding to B_1 , $\mu_{\text{rms}}^{\text{S1,fit}}/\mu_{\text{rms}}^{\text{N1,fit}} = 1.002$, where $\mu_{\text{rms}}^{\text{S1,fit}}$ is the Gaussian mean of the DI distribution and they overlap to a large extent in the second measurement corresponding to B_2 , $\mu_{\text{rms}}^{\text{S2,fit}}/\mu_{\text{rms}}^{\text{N2,fit}} = 1.1$. As can be seen in the figure, the relative shift between the DI and noise distributions observed in the second measurement with B_2 , falls well within the standard deviation of the first measurement with B_1 . Moreover, the larger shift observed in the second measurement for B_2 can be attributed to nonidealities of the measurement, such as the accuracy of the manual setting of the relative pump phase and the input powers of the three beams as well as the phase stability of the three generators over time which sets a limit on the amount of averaging which can be applied in each case.

Furthermore, it is important to note that varying the resolution bandwidth of the measurement mainly changes the noise floor of system but has no effect on the received output power originating from the coherent signals applied in the

interference experiment. Hence, by calculating the ratio of the average power received in the destructive interference experiment measured with B_1 (B_2) to the average power received in the constructive interference experiment (not shown in the figure) we get a value of -25 dB (-37.9 dB) which corresponds to a coherent cancellation of 99.7% (99.98%) of input signal and idler photon fluxes.

V. COMPARISON WITH A MICROWAVE MIXER

In this section, we compare our Josephson junction 3-wave beam-splitter/combiner to a diode based microwave mixer. A microwave mixer has three ports a local oscillator (LO), a radio frequency (RF) and an intermediate frequency (IF) ports. An ideal mixer mixes two input signals with different frequencies LO and IF or LO and RF and generates an output signal at the RF or IF ports which consists of the sum and difference frequencies of the two inputs. This frequency conversion property of the device makes it invaluable component for wireless and cellular communication systems, such as transmitters and receivers which employ modulation (up-conversion) and demodulation (down-conversion) schemes. Mixers are also widely used in homodyne and heterodyne microwave experiments and in measuring devices such as vector network analyzers and spectrum analyzers. Microwave mixers achieve frequency conversion using nonlinear elements such as Schottky diodes or field effect transistors (FETs) which effectively function as nonlinear resistors [7]. However, the nonlinearity of the diodes and transistors can also generate other high order harmonics and products of the input frequencies. Hence, a considerable engineering effort is made to impedance match the three ports and filter undesired products by using reactive or resistive elements at the different ports.

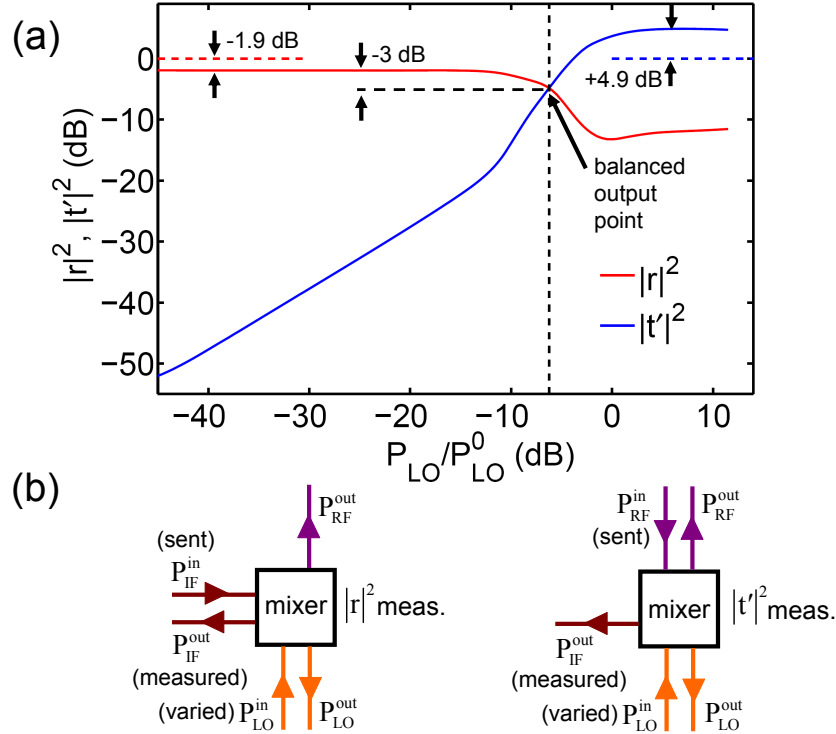


FIG. S4: (colored online). Reflection and conversion measurements of a microwave mixer. (a) The reflection parameter $|r|^2$ (red curve) and the conversion parameter $|t'|^2$ (blue curve) are plotted in log-log scale as a function of the normalized local oscillator (LO) power P_{LO}/P_{LO}^0 , where $P_{LO}^0 = 6.5$ dBm is the LO power at which $|r|^2$ is minimum. Both measurements were taken at the IF port. In the reflection measurement, a coherent tone at f_{IF} was applied to the IF port with input power P_{IF}^{in} . In the conversion measurement, a coherent tone at f_{RF} was applied to the RF port with input power P_{RF}^{in} , which is set to yield at the balanced output point the same IF output power as the one measured in reflection for P_{IF}^{in} . The vertical dashed line indicates the balanced output point of the mixer at the IF port. (b) A device cartoon showing the signals employed in the reflection and conversion measurements of the mixer.

In the control experiment shown in Fig. S4, we measured a commercial off the shelf triple-balanced mixer by Marki

Microwave made of Schottky diodes. The measured mixer has a wide-bandwidth 2 – 20 GHz for the RF and LO ports and 1 – 10 GHz for the IF port. For simplicity, we choose to work with similar frequencies as in the JPC experiment where the LO, RF and IF play the role of the pump $f_{LO} = 6.285$ GHz, the idler $f_{RF} = 14.687$ GHz and the signal $f_{IF} = 8.402$ GHz respectively.

One of the figures of merit of mixers is conversion loss, which is defined as $L_{\text{conv}}(\text{dB}) = 10 \log(P_{RF}^{\text{in}}/P_{IF}^{\text{out}})$, where P_{RF}^{in} is the RF input power and P_{IF}^{out} is the converted power at the IF port. A dual figure of merit can also be defined for the IF input and the RF output. The main loss mechanisms which contribute to L_{conv} are, resistive loads added to the ports in order to attenuate undesired products, internal losses of the diodes or FETs, and power conversion to other harmonics or generated products. It is worthwhile mentioning that the magnitude of conversion loss is also strongly dependent on the applied LO power.

In Fig. S4 (a) we plot the reflection ($|r|^2 = P_{IF}^{\text{in}}/P_{IF}^{\text{in}}$) and the conversion ($|t'|^2 = P_{IF}^{\text{out}}/P_{RF}^{\text{in}}$) parameters of the microwave mixer using red and blue curves respectively as a function of the normalized LO power P_{LO}/P_{LO}^0 , where P_{LO}^0 is the LO power at which $|r|^2$ is minimum. The reflection and conversion parameters are plotted in log-log scale in order to cover the several orders of magnitude spanned by the data. The signals employed in each measurement are sketched in panel (b). It is important to note that unlike the JPC, in a microwave mixer, due to the reflection and conversion losses, the sum $|r|^2 + |t'|^2$ (in linear units) is not equal to one as can be seen in the figure. The measurement was taken using spectrum analyzer centered at the IF frequency in zero frequency span mode. We also used a circulator on the IF port of the mixer in order to separate input and output signals. The input power of the IF tone used in the reflection measurement is $P_{IF}^{\text{in}} = -43.07$ dBm. The 1.9 dB loss indicated in the figure represents the reflection loss of the IF tone when the LO is turned off ($P_{IF0}^{\text{out}}/P_{IF}^{\text{in}}$). From the reflection measurement result we find the balanced signal output point of the device, namely $P_{LO}^{\text{in}} = 0.57$ dBm at which the reflected signal P_{IF}^{out} drops by 3 dB relative to P_{IF0}^{out} . For the conversion measurement, we used an RF tone with input power $P_{RF}^{\text{in}} = -28.92$ dBm. This input power was set to yield the same P_{IF}^{out} (-47.97 dBm) measured at the IF port in reflection at the “50/50 beam-splitting” point. Using the definition of conversion loss for mixers we find $L_{\text{conv}} = 19.05$ dB at the “50/50 beam-splitting” point. As can be seen in the figure, for high LO powers for example at 15 dBm the conversion loss improves by about 9.8 dB and becomes $L_{\text{conv}} = 9.25$ dB which matches the typical values listed in the data sheet of the mixer.

Furthermore, we can place a lower bound on the inherent power conversion loss of the mixer which we define as $L'_{\text{conv}} = L_{\text{conv}} - L_r^{RF} - L_r^{IF}$, where L_{conv} is the conversion loss at the optimal LO power at 15 dBm and L_r^{RF} , L_r^{IF} are the measured reflection losses at the IF (1.9 dB) and RF (1.6 dB) ports at f_{IF} and f_{RF} respectively. This lower bound L'_{conv} which equals to 5.7 dB in the measured mixer shows that it is not possible to model a microwave mixer made of Schottky diodes using a simple model of a lossless mixer with the addition of attenuations on the different ports.

VI. DEVICE AND MEASUREMENT DETAILS

The resonators of the JPC are implemented using Nb over a 430 μm thick sapphire substrate with a 2 μm thick silver ground plane evaporated on the back side, which enhances thermalization and microwave control. The Nb layer is patterned using a standard photolithography step and etched using reactive ion etching. The JRM of the device is incorporated using a standard e-beam lithography process followed by two angle shadow evaporation of aluminum (with an oxidation step in between) and lift off. The Josephson junctions of the JRM are nominally identical with critical current $I_0 = 3 \pm 0.5 \mu\text{A}$. The Josephson junction area is $5 \mu\text{m} \times 1 \mu\text{m}$, while the loop area of the JRM is about $50 \mu\text{m}^2$. A large overlap area (partially shown in Fig. 2 (b)) is established between the Nb part of the resonators and the Al wires of the JRM which is preceded by plasma cleaning. No observable losses were measured in our samples due to this interface. In the experiment, the JRM is biased with half a flux quantum using an external magnetic coil attached to the copper box housing the device.

The measurements were taken in a dilution fridge at a base temperature of 30 mK. The experimental setup is similar to the one used in Ref. [8]. It consists of three input lines for the signal, idler and pump, which are composed of stainless steel semi-rigid coax cables with attenuators at the 4 K and mixing-chamber stages. The input lines of the signal and idler are connected to the device ports, shown in Fig. 2 (a), through circulators at base which separate between input and output signals of the system. The third port of the circulators is connected to two output lines for the signal and idler. Each output line consists of: (1) two isolators in series at the mixing-chamber stage with bandwidth 4 – 12 GHz followed by a low-pass filter with a cutoff frequency at 12 GHz to partially protect the sample from out-of-band noise of the HEMT, (2) a short semi-rigid coax cable of NbTi which connects between the mixing-chamber and the 4 K stages, (3) a HEMT amplifier at the 4 K plate with bandwidth 4 – 8 GHz on the signal side and 4 – 12 GHz on the idler side, (4) room temperature amplifiers and coax cables. The lack of commercial cryogenic

microwave circulators and isolators and HEMT amplifiers which work around 15 GHz makes it difficult to measure directly the idler output power of our device.

The interference fringes shown in Figs. 4, 5 and 6 are measured using time traces of a spectrum analyzer in zero frequency span mode with center frequency f_S , resolution bandwidth and video bandwidth of 510 Hz. The pump phase was varied as a function of time by offsetting the frequency $f_P = f_I - f_S$ on the pump generator by a few Hz. The frequency offset Δf was set to be much less than the resolution bandwidth of the measurement in order to combine the reflected signal at f_S and the converted idler at $f_S - \Delta f$ in a phase-sensitive manner. Similar interference fringes can be obtained by alternatively offsetting f_S or f_I instead.

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